EXPONENTIAL FUNCTIONS

REVIEW PACKET FOR UNIT TEST

TOPICS OF STUDY:

- Recognizing Exponential Functions from Equations, Graphs, and Tables
- Graphing Exponential Functions Using a Table of Values
- Identifying the Domain, Range, and Horizontal Asymptote from a Graph
- Transforming Exponential Functions
- Average Rate of Change
- Linear-Exponential Systems
- Finding the Common Ratio from a Table
- Geometric Sequences
- Exponential Growth and Decay, Including Compound Interest

MEMORIZE:

General Form of an Exponential Function
\[ y = a \cdot b^{x-h} + k \]

Exponential Growth:
\[ y = a(1 + r)^n \]

Exponential Decay:
\[ y = a(1 - r)^n \]

Compound Interest:
\[ A = P \left(1 + \frac{r}{n}\right)^{nt} \]

Explicit Rule for Geometric Sequences
\[ a_n = a_1 \cdot r^{n-1} \]
1) Which of the following is an exponential function?

(a) \( y = -13x^2 + 2 \)  
(b) \( y = 2^x \)  
(c) \( y = 7x - 3 \)  
(d) \( y = |x + 4| \)

2) Which graph represents the exponential decay of a radioactive element?

(a)  
(b)  
(c)  
(d) 

3) Which transformation best describes the relationship between the functions \( f(x) = 2^x \) and \( g(x) = \left(\frac{1}{2}\right)^x \)?

(a) reflection in the line \( y = x \)  
(b) reflection in the origin  
(c) reflection in the \( x \)-axis  
(d) reflection in the \( y \)-axis

4) Which equation below represents exponential decay?

(a) \( y = 3^x \)  
(b) \( y = 1.4^x \)  
(c) \( y = \left(\frac{3}{2}\right)^x \)  
(d) \( y = 0.5^x \)
5) Find the range of $y = 2^x - 1$.

(a) all real numbers  
(b) $y > 0$  
(c) $y > -1$  
(d) $y \geq -1$

6) State the equation of the horizontal asymptote of $y = 2^x - 4$.

(a) $x = 0$  
(b) $x = -4$  
(c) $y = 0$  
(d) $y = -4$

7) The graph of the equation $y = -4^x$ lies in quadrants

(a) III and IV  
(b) II and III  
(c) I and IV  
(d) I and II

8) Which equation represents the relationship between $x$ and $y$ in the table below?

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>5</td>
<td>10</td>
<td>20</td>
<td>40</td>
<td>80</td>
<td>160</td>
<td>320</td>
</tr>
</tbody>
</table>

(a) $y = 2^x$  
(b) $y = 2x + 5$  
(c) $y = 5(2^x)$  
(d) $y = 2x$

9) The graph of the equation $y = 3^x$ is shifted two units to the right. The new equation is:

(a) $y = 3^x + 2$  
(b) $y = 3^{x-2}$  
(c) $y = 3^x + 2$  
(d) $y = 3^x - 2$

10) Which of the following equations could represent the graph below?

(a) $y = 1.5^x + 1$  
(b) $y = 1.5^{x+1}$  
(c) $y = 1.5^{x-1}$  
(d) $y = 1.5^x - 1$
11) Identify the graph of $y = -4(2)^x$.

12) What is the approximate solution to the given system?

$$\begin{align*}
  f(x) &= 2^x \\
  g(x) &= -4x + 10
\end{align*}$$

(a) $(1.69, 3.23)$  
(b) $(1.67, 3.33)$  
(c) No solution  
(d) $(3.23, 1.69)$

13) Which of the following functions represents a vertical shrink?

(a) $f(x) = 5 \cdot 3^x$  
(b) $g(x) = \left(\frac{1}{2}\right) \cdot 4^x$

(c) $h(x) = \left(\frac{1}{2}\right) \cdot x^2$  
(d) $j(x) = 2 \cdot \left(\frac{1}{4}\right)^x$
14) Ismael plans to purchase a car that depreciates (loses value) at a rate of 14% per year. The initial cost of the car is $21,000. Which equation represents the value, $v$ of the car after 3 years?

(a) $v = 21000(0.14)^3$
(b) $v = 21000(0.86)^3$
(c) $v = 21000(1.14)^3$
(d) $v = 21000(1.86)^3$

15) Is the equation $A = 1500(1 - 0.35)^t$ a model of exponential growth or exponential decay, and what is the rate (percent) of change per time period?

(a) exponential growth and 35%
(b) exponential growth and 65%
(c) exponential decay and 35%
(d) exponential decay and 65%

16) A radioactive substance has an initial mass of 100 grams and its mass halves every 4 years. Which expression shows the number of grams remaining after $t$ years?

(a) $100(4)^\frac{t}{4}$
(b) $100(4)^{-2t}$
(c) $100\left(\frac{1}{2}\right)^\frac{t}{4}$
(d) $100\left(\frac{1}{2}\right)^{4t}$

17) Which statement about this sequence is true?

5, -25, 125, -625, …

(a) The sequence is not geometric and the common ratio is -5.
(b) The sequence is arithmetic and the common difference is -5.
(c) The sequence is geometric and the common ratio is 5.
(d) The sequence is geometric and the common ratio is -5.

18) True or false? The following table represents an exponential function.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>0.25</td>
<td>0.5</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>
19) True or false? The following table represents an exponential function.

<table>
<thead>
<tr>
<th>x</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
</tr>
</tbody>
</table>

20) Find the average rate of change of the function \( f(x) = 3^x + 2 \) over the interval \(-2 \leq x \leq 2\).

21) Find the average rate of change of the function below over the interval \(2 \leq x \leq 6\).

<table>
<thead>
<tr>
<th>x</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>5</td>
<td>25</td>
<td>125</td>
<td>625</td>
</tr>
</tbody>
</table>

22) Find the average rate of change of the function below over the interval \(0 \leq x \leq 2\).

23) Describe all of the transformations of the function \( f(x) = -\frac{1}{3}(4)^{x+1} + 3 \) as compared to the parent function \( f(x) = 4^x \).
24) Ms. Java bought a piece of property upstate for $35,000. Each year, it appreciates (gains value) at a rate of 5%.

a) What will the total value of the property be at the end of the 4th year?

b) How much did the property increase in value by the end of the 4th year?

25) A used car was purchased in July 2006 for $15,900. If the car depreciates 13% of its value each year, what is the value of the car, to the nearest hundred dollars, in July 2014?

26) The Sullivans inherited $20,500, which they want to invest for their child’s future college expenses. If they invest it at an interest rate of 4.75%, with interest compounded quarterly, determine the value of the account, in dollars, after 5 years. Round to the nearest cent.

27) Consider the geometric sequence 4, -8, 16, -32, 64….

Part A: Write the explicit rule for the sequence.

Part B: Use the explicit rule to find the 10th term of the sequence.
28) The table below shows a car’s value for 3 years after it was purchased. The values form a geometric sequence. Write the explicit rule for this sequence.

<table>
<thead>
<tr>
<th>Year</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$18,000</td>
</tr>
<tr>
<td>2</td>
<td>$15,300</td>
</tr>
<tr>
<td>3</td>
<td>$13,005</td>
</tr>
</tbody>
</table>

29) Consider the following exponential function, \( y = 4(0.25)^x \)

a) Graph the exponential function by creating a table of values for the interval \(-1 \leq x \leq 3\). (Draw the axes such that all values will fit on this grid.)

b) What is the domain? _____________________________________________________

c) What is the range? _____________________________________________________

d) What is the horizontal asymptote? ________________________________________
30) On the axes below, graph $y = 2^{x+1} - 3$.

a) What is the domain? _________________________________________________

b) What is the range? ________________________________________________

c) What is the horizontal asymptote? ____________________________________

31) Graph the following system of equations and state the coordinates of all solutions.

\[
\begin{align*}
    y &= -2x + 4 \\
    y &= \left(\frac{1}{2}\right)^x + 4
\end{align*}
\]